UDC 534.213

THE EFFECT OF TURBULENT NOISE ON A PRESSURE RECEIVER LOCATED IN AN ELASTIC MEDIUM

PMM Vol.40, № 4, 1976, pp.746-751
L. A. SHENIAVSKII and V. I. SHMAL'GAUZEN
(Moscow)
(Received December 20, 1974)

Correlation characteristics of the perturbation field in an elastic medium whose surface is subjected to turbulent pressure pulsations are calculated. Spectral density of the noise signal received by a flat receiver located under an elastic layer of adequate thickness is determined.

In investigations of turbulence noise it is often necessary to locate the noise receiver under layer of elastic material. The computation of specific systems presents considerable difficulties. Qualitative results may be obtained by analyzing the solution of the problem of a random signal propagation in an elastic half-space, which is induced in the plane bounding that half-space. If it is assumed that the receiver located in an elastic medium does not disturb the acoustic field, it is possible to estimate also the spectrum of the perceived signal.

1. We introduce a rectangular system of Cartesian coordinates $OXYZ$ whose plane $z = 0$ coincides with the half-space boundary, and the oz-axis directed along the outward normal to the half-space. Let pressure p at the boundary be a steady and steadily-connected random quantity whose space-time spectrum $S_p(\omega, k_x, k_y)$ and, consequently, also the correlation function $R_p(\tau, \xi, \eta)$ are known. We assume that the plane of the hydrophone is perpendicular to the oz-axis and that the hydrophone reacts only to the component of stress σ_{zz} that is normal to its surface, so that for a determinate action the stress at the hydrophone is

$$V = \int_{s_h} \gamma_v(\mathbf{r})\, \sigma_{zz}(\mathbf{r})\, ds \tag{1.1}$$

where γ_v is the electromechanical coefficient of the hydrophone and s_h the area of the hydrophone.

The relation between the space-time spectra S_p and S_σ of the random quantities p and σ_{zz} is of the form

$$S_\sigma(\omega, k_x, k_y, z) = |L(\omega, k_x, k_y, z)|^2\, S_p(\omega, k_x, k_y) \tag{1.2}$$

where $(L(\omega, k_x, k_y, z)$ is the medium transfer function. Since the plane of the hydrophone is normal to oz, its insertion depth $|z|$ in the elastic medium is a parameter of function L. We define the space-time correlation function of quantity σ_{zz} as follows:

$$R_\sigma(\tau, \xi, \eta) = \frac{1}{8} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_\sigma(\omega, k_x, k_y) \exp\, [i(\omega\tau + k_x\xi + K_y\eta]\, d\omega\, dk_x\, dk_y \tag{1.3}$$

Using (1.1) and (1.3) for the autocorrelation function of signal $R_v(\tau)$ on the hydrophone we obtain

$$R_v(\tau) = \int_{s_h} \int_{s_h'} \gamma_v(\mathbf{r})\, \gamma_v(\mathbf{r}')\, R_\sigma(\mathbf{r} - \mathbf{r}', \tau)\, ds_h' ds_h \tag{1.4}$$

Spectrum of the signal received by the hydrophone can be obtained by applying the inverse Fourier transformation to (1.4).

To determine function L we shall consider the case when the action at the boundary is of the form

$$\sigma_{zz}|_{z=0} = -p_{z0}\exp[i(\omega t + k_x x + k_y y)] \qquad (1.5)$$
$$\sigma_{zx}|_{z=0} = \sigma_{zy}|_{z=0} = 0$$

Using conventional procedures from the equations of motion of the elastic medium [1, 2] we obtain

$$\sigma_{zz} = -\frac{p_{z0}}{\Delta}[(\omega^2 - 2c_t^2k_0^2)(k_0^2 - k_{tz}^2)\exp(ik_{lz}z) - 4c_t^2k_0^2k_{tz}k_{lz}\exp(ik_{tz}z)]\exp[i(k_x x + k_y y + \omega t)] \qquad (1.6)$$
$$k_0^2 = k_x^2 + k_y^2$$
$$\Delta = c_l^2k_l^2(k_0^2 - k_{tz}^2) - 2c_t^2k_0^2(k_0^2 - k_{tz}^2 + 2k_{lz}k_{tz})$$

where c_l and c_t are velocities of longitudinal and transverse waves, respectively.

The dispersion formulas are of the form

$$k_l^2 = k_{lz}^2 + k_0^2 = \omega^2 / c_l^2, \qquad k_t^2 = k_{tz}^2 + k_0^2 = \omega^2 / c_t^2$$

The equality of Δ to zero corresponds to the dispersion equation for surface waves (Rayleigh waves).

In the case of "rubberlike" media which are commonly used for the protection of receivers, the transverse wave becomes strongly attenuated, hence, if $|z|$ is not too small, it is possible to restrict the analysis to longitudinal waves only. In such approximation the transfer function L can be represented in the form

$$L(\omega, k_0) = L_0(\omega, k_0)\exp(i\sqrt{\omega^2/c_l^2 - k_0^2}\, z) \qquad (1.7)$$
$$L_0 = (2k_0^2 - \omega^2/c_t^2)^2[(2k_0^2 - \omega^2/c_t^2)^2 + 4k_0^2 \times \sqrt{(\omega^2/c_l^2 - k_0^2)(\omega^2/c_t^2 - k_0^2)}\,]^{-1}$$

If the random pressure on the surface is isotropic, its correlation function depends on $\rho = \sqrt{\xi^2 + \eta^2}$, and the spectral density on k_0. Substituting (1.2) into (1.3), exchanging variables, and passing to the Fourier-Bessel transformation, we obtain

$$R_\sigma(\tau, \rho) = \frac{\pi}{4}\int_{-\infty}^{\infty}\exp(i\omega\tau)\int_0^{\infty}|L(\omega, k_0, z)|^2 \times S_p(\omega, k_0)J_0(k_0\rho)k_0\,dk_0d\omega \qquad (1.8)$$

In the wave zone where $|z|$ is fairly high, the region of integration with respect to k_0 can be limited to ω / c_l, since for considerable k_0 function $|L|^2$ rapidly decreases. Moreover, as shown by the analysis of the formula for L_0 in (1.7), $|L_0|^2 \approx 1$ when $c_l / c_t \gg 1$ and $0 \leqslant k_0 \leqslant \omega / c_l$. It is known that for all media $c_l / c_t > \sqrt{2}$. When $c_l / c_t \geqslant 4$, then in the considered interval of k_0 the deviation of $|L_0|^2$ from unity does not exceed 6% and for $c_l / c_t \geqslant 10$ it does not exceed 0.35%.

Thus, when $c_l / c_t \gg 1$ it is possible to represent approximately formula (1.8) as follows:

$$R_\sigma(\tau, \rho) = \frac{\pi}{4}\int_{-\infty}^{\infty}\exp(i\omega\tau)\frac{\omega^2}{c_l^2}\int_0^1 S_p(\omega, \varkappa)J_0\left(\rho\frac{\omega}{c_l}\varkappa\right)\varkappa d\varkappa d\omega$$
$$(\varkappa = c_l k_0/\omega)$$

A similar result was obtained in [3] for the acoustic case.

Note that when $0 \leqslant k_0 \leqslant \omega / c_l$, formula (1.6) implies that the effect of the transverse wave on σ_{zz} compared to that of the longitudinal wave is of order of smallness of $(c_l / c_t)^{-3}$. This justifies the previous statement about neglecting the transverse wave.

2. Let us consider the case of isotropic turbulence. We use the following approximation for the space-time correlation function:

$$R_p(\tau, \rho) = R_0 \exp[-(\beta|\tau| + \alpha\rho)]$$

which yields for the spectral density the formula

$$S_p(\omega, k_0) = S_0 \alpha\beta (\beta^2 + \omega^2)^{-1} (\alpha^2 + k_0^2)^{-3/2}, \qquad S_0 = 4R_0 / \pi^2 \tag{2.1}$$

Substituting (2.1) into (1.3) and taking into account that the integrands are even with respect to ω, we obtain

$$R_\sigma(\tau, \rho) = \frac{2R_0\alpha\beta}{\pi c_l^2} \int_0^\infty \frac{\omega^2 \cos(\omega\tau)}{\beta^2 + \omega^2} \int_0^1 \frac{J_0(\rho\omega c_l^{-1}\varkappa)\varkappa d\varkappa}{(\alpha^2 + \omega^2 c_l^{-2}\varkappa^2)^{3/2}} d\omega \tag{2.2}$$

The dispersion of σ_{zz} on the wave zone is $R_\sigma(0, 0)$. Carrying out computations by formula (2.2), we obtain

$$\frac{R_\sigma(0, 0)}{R_0} = 1 - \frac{2\gamma}{\pi} \begin{cases} (\gamma^2 - 1)^{-1/2} \operatorname{arctg} \sqrt{\gamma^2 - 1}, & \gamma > 1 \\ (1 - \gamma^2)^{-1/2} \ln[(1 + \sqrt{1 - \gamma^2})\gamma^{-1}], & \gamma < 1 \end{cases} \tag{2.3}$$

$$\gamma = \alpha c_l / \beta$$

The quantities $R_\sigma(\tau, 0)$ and $R_\sigma(0, \rho)$ are fairly easily obtained from (2.2). We have

$$R_\sigma(\tau, 0) = R_p(\tau, 0) - R_0 \frac{2\alpha\beta}{\pi} \int_0^\infty \frac{\cos(\omega\tau)\, d\omega}{(\beta^2 + \omega^2)\sqrt{\alpha^2 + \omega^2 c_l^{-2}}} \tag{2.4}$$

$$R_\sigma(0, \rho) = R_p(0, \rho) - R_0 \frac{2\alpha}{\pi} \int_0^\infty \frac{\operatorname{arctg}(c_l\beta^{-1}k_0)\, J_0(\rho k_0)}{(\alpha^2 + k_0^2)^{3/2}} k_0 dk_0$$

The introduction of dimensionless time $\theta = \beta\tau$ and coordinate $x = \alpha\rho$ reduces formula (2.4) to dependence of the single parameter $\gamma = \alpha c_l / \beta$. The numerically computed relationships $R_\sigma(\theta, 0) / R_\sigma(0, 0)$ and

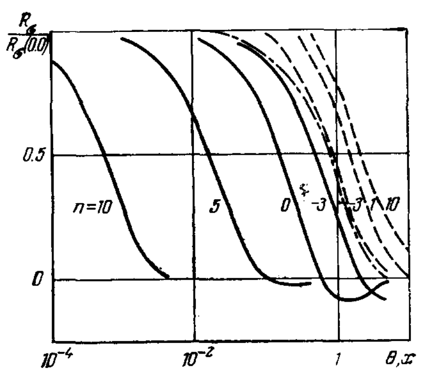

Fig. 1

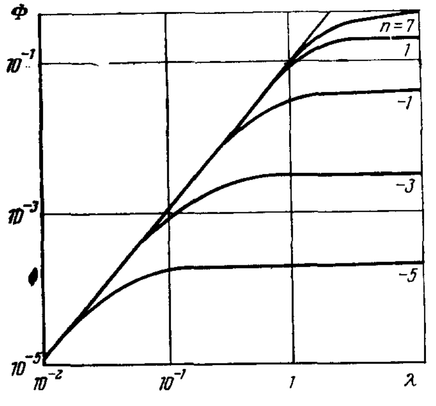

Fig. 3

$R_\sigma(0, x) / R_\sigma(0, 0)$ are shown in Fig. 1 in terms of θ and x by solid and dash lines, respectively, for several values of $\gamma = 2^n$. The curve of e^{-x} is shown there by the dash-dot line.

These curves indicate that in the wave zone the correlation time diminishes and the correlation radius increases. These changes are more pronounced the greater γ.

Let us determine the spectrum of signal received by the hydrophone in an idealized case. The receiver is assumed to be a circle of radius r_0 with ideal acoustic insulation, and its sensitivity to be constant over its surface. If we denote the point of stress application by M, the quantity γ_v in (1.1) is defined as follows:

$$\Upsilon_v = \begin{cases} \Upsilon_v = \text{const}, & M \subset s_h \\ 0, & M \not\subset s_h \end{cases} \tag{2.5}$$

where s_h is the surface of the hydrophone.

We substitute the expression for γ_v into (1.4) and taking into account the noise isotropy, pass to polar coordinates. Computations similar to those in [4] yield

$$R_v(\tau) = 4\pi^2 \Upsilon_v^2 r_0^2 \int_0^{2r_0} R_\sigma(\tau, \rho) K(\rho) \rho d\rho \tag{2.6}$$

$$K(\rho) = \arccos \frac{\rho}{2r_0} - \frac{\rho}{2r_0} \sqrt{1 - \left(\frac{\rho}{2r_0}\right)^2}$$

We substitute (2.2) into (2.6) and apply to the obtained expression the Fourier transformation. We represent the spectrum of signal on the hydrophone as follows:

$$S_v(\omega) = 16 (\pi r_0^2)^2 \Upsilon_v^2 S_0 \frac{\mu\beta}{\beta^2 + \omega^2} \int_0^{2\lambda} \frac{F(\eta)\, \eta\, d\eta}{[(2\mu)^2 + \eta^2]^{3/2}} \tag{2.7}$$

$$\lambda = r_0\omega / c_l, \qquad \mu = r_0\alpha$$

$$F(\eta) = \int_0^1 J_0(\eta\xi)(\arccos \xi - \xi \sqrt{1 - \xi^2})\, \xi\, d\xi = \pi\eta^{-2} \left[J_1\left(\frac{\eta}{2}\right)\right]^2$$

In the case of a point receiver, i.e. for $\mu \to 0$, the integral in (2.7) increases as $1/\mu$, hence $S_v(\omega)$ remains finite. For defining the spectrum of the signal received by the hydrophone it is convenient to use the function of two variables

$$\Phi(\lambda, \mu) = \mu^3 \int_0^\lambda \frac{[J_1(\xi)]^2\, d\xi}{\xi(\mu^2 + \xi^2)^{3/2}} \tag{2.8}$$

It can be readily verified that for $\mu \ll 1$ and $\lambda \to \infty$ we have $\Phi \to \mu^2 / 4$. If, however, $\mu \gg \lambda$, then for function $\lambda \to \infty$, Φ depends only slightly on parameters, and at the limit tends to $1/2$. Curves of Φ computed by formula (2.8) are shown in Fig. 2 in terms of λ for several values of $\mu = 2^\mu$. Taking into account (2.8) we can represent formula (2.7) in the form

$$S_v(\omega) = 2\pi(\pi r_0^2) \Upsilon_v^2 \frac{S_0\beta}{\alpha^2(\beta^2 + \omega^2)} \Phi\left(\frac{r_0\omega}{c_l}, r_0\alpha\right) \tag{2.9}$$

We determine the dispersion of the signal received by the hydrophone by formula (2.6) setting $\tau = 0$. Substituting (2.6) into formula (2.2) and carrying out necessary computations, we obtain

$$D_v \doteq R_v(0) = 2\pi (\pi r_0^2)^2 \gamma_v^2 S_0 \varphi(r_0\alpha, \gamma)$$

$$\varphi(\mu, \gamma) = \mu \int_0^\infty \frac{[J_1(\xi)]^2 \operatorname{arctg}(\xi\gamma/\mu)}{\xi(\mu^2+\xi^2)^{3/2}} d\xi$$

Curves of the set of functions $\varphi(\gamma)$ obtained by these computations are shown in Fig. 3 for several vaues of parameter $r_0\alpha = 2^n$.

3. Let us consider the more general case, when the turbulent pressure at the boundary is not isotropic. The direction coincident with the stream velocity and perpendicular to it is usually selected. In such case it is more convenient to represent (1.4) in the form

$$R_v(\tau) = \int_{s_h}\int_{s_h'} \gamma_v(x_1, y_1)\gamma_v(x_1+\xi, y_1+\eta) R_\sigma(\xi, \eta, \tau)\, dx_1\, dy_1 d\xi\, d\eta$$

Let us consider the function that defines the hydrophone effect

$$\Theta(\xi, \eta) = \int_{s_h} \gamma_v(x_1, y_1)\gamma_v(x_1+\xi, y_1+\eta)\, dx_1\, dy_1 \qquad (3.1)$$

Applying to $R_v(\tau)$ the Fourier transformation and allowing for (3.1), we obtain

$$S_v(\omega) = \int_{s_h} \Theta(\xi, \eta)\,\Gamma(\xi, \eta, \omega)\, d\xi\, d\eta \qquad (3.2)$$

$$\Gamma(\xi, \eta, \omega) = \frac{1}{4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_\sigma(\omega, k_x, k_y) \exp[i(k_x\xi + k_y\eta)]\, dk_x\, dk_y$$

which represents the relative spectral density of σ_{zz} in the receiver plane. Since function γ_v outside region s_h is zero, the integral over the surface s_h can be replaced by an integral with infinite limits.

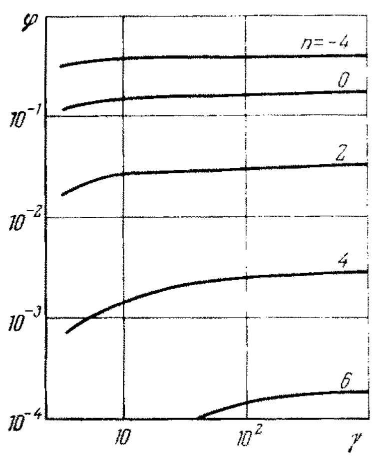

Fig. 3

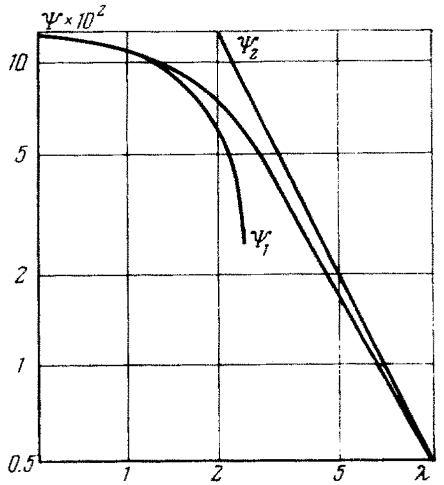

Fig. 4

Allowing for (1.2) from (3.2) we have

$$S_v(\omega) = \frac{1}{4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_\theta(k_x, k_y) S_p(k_x, k_y, \omega) \mid L(k_x, k_y, \omega, z) \mid^2 dk_x dk_y \qquad (3.3)$$

$$\Phi_\theta(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Theta(\xi, \eta) \exp [i(k_x\xi + k_y\eta)] d\xi d\eta$$

According to [5] a turbulent stream generates at the boundary a pressure whose relative spectral density can be represented as follows:

$$\Gamma_0(x, y, \omega) = S_\omega(\omega) \exp [-(\alpha |x| + \beta |y|) \omega / V_c] \times \cos (\omega x / V_c) \qquad (3.4)$$

where V_c is the convective velocity. The space-time spectrum of pressure at the boundary S_p in (3.4) is of the form

$$S_p(k_x, k_y, \omega) = \frac{1}{\pi} \frac{S_\omega(\omega) \alpha\beta\omega^2 / V_c^2}{(\beta\omega / V_c)^2 + k_y^2} \times \qquad (3.5)$$

$$\left[\frac{1}{(\alpha\omega / V_c)^2 + (\omega / V_c + k_x)^2} + \frac{1}{(\alpha\omega / V_c)^2 + (\omega / V_c - k_x)^2} \right]$$

To obtain the signal spectrum at the hydrophone located in the wave zone, where $k_x < \omega / c_l$ and $k_y < \omega / c_l$, we substitute (3.5) and (1.7) into (3.3). According to [5] the convective velocity varies within the limits $0.6\, V_0 < V_c < V_0$, where V_0 is the stream velocity. Usually $V_0 \ll c_l$, hence terms of order $(V_c / c_l)^2$ can be neglected in the product $S_p |L|^2$.

The space-time spectral density in the wave zone is obtained by substituting (3.4) and (1.7) into (1.2). Taking into consideration what was said above, we obtain the approximate formula

$$S_\sigma(\omega, k_0) \cong \begin{cases} \dfrac{2\alpha V_c^2 S_\omega(\omega)}{\pi\beta(1+\alpha^2)\omega^2}, & k_0 < \omega / c_l \\ 0, & k_0 > \omega / c_l \end{cases} \qquad (3.6)$$

The signal spectrum at the hydrophone can be presented, with (3.3) taken into account, in the form

$$S_v(\omega) = \frac{\alpha V_c^2 S_\omega(\omega)}{2\pi\beta(1+\alpha^2)\omega^2} \iint_{\sigma_1} \Phi_\theta(k_x, k_y) dk_x dk_y \qquad (3.7)$$

where σ_1 is a circle of radius ω / c_l. In the case of the idealized hydrophone whose sensitivity is defined by formula (2.5), function $\Theta(\xi, \eta)$ depends on $\rho = \sqrt{\xi^2 + \eta^2}$. Hence further computation in which formulas (3.1), (3.3) and (3.7) are used, are similar to those used for deriving formula (2.9) for $S_v(\omega)$ in the case of an isotropic pressure field. After necessary computations we obtain

$$S_v(\omega) = 4(\pi r_0^2)^2 \gamma_v^2 \frac{\alpha}{\beta(1+\alpha^2)} \left(\frac{V_c}{c_l}\right)^2 S_\omega(\omega) \Psi\left(\frac{r_0\omega}{c_l}\right), \quad \Psi(\lambda) = \frac{1}{\lambda^2} \int_0^\lambda \frac{[J_1(\xi)]^2}{\xi} d\xi$$

It can be readily shown that for $\lambda \to 0$

$$\Psi \approx \Psi_1 = 1/8\,(1 - 1/8\,\lambda^2)$$

When $\lambda \to \infty$, then $\Psi \approx \Psi_2 = \lambda^2 / 2$. Function $\Psi(\lambda)$ is shown in Fig. 4.

Since for $V_0 \ll c_l$ the field σ_{zz}, as implied by (3.6), is isotropic, formula (1.8) can be used for obtaining its correlation function. If in (3.4) function $S_\omega(\omega)$ is even, then by substituting (3.6) into (1.8), we obtain

$$R_\sigma(\tau, \rho) = \frac{\alpha V_c^2}{\beta(1+\alpha^2)c_l\rho}\int_0^\infty \frac{S_\omega(\omega)}{\omega} J_1\left(\frac{\omega\rho}{c_l}\right)\cos(\omega\tau)\,d\omega$$

The dispersion of quantity σ_{zz} can be found from the last formula by setting in it $\tau = 0$ and passing to the limit for $\rho \to 0$. We have

$$R_\sigma(0,0) = \frac{\alpha}{2\beta(1+\alpha^2)}\left(\frac{V_c}{c_l}\right)^2 \int_0^\infty S_\omega(\omega)\,d\omega$$

The obtained results have a simple physical interpretation. The layer of elastic medium lying over the receiver is an additional filter which transmits only those components of the external random field of pressures which satisfy the inequality $\omega > k_0 c_l$. This results in further suppression of low-frequency perturbations, as compared to the case when the receiver is directly subjected to a turbulent flow.

REFERENCES

1. Landau, L. D. and Lifshits, E. M., Theory of Elasticity. Theoretical Physics. Vol. 7, (English translation), Pergamon Press, Book № 13128, 1959.
2. Leibenzon, L. S., Course of Theory of Elasticity. Moscow-Leningrad, Gostekhizdat, 1947.
3. Kur'ianov, B.F., Space correlation of fields radiated by random forces in a plane. Akust. Zh., Vol. 9, № 4, 1963.
4. Corcos, G. M., The structure of the turbulent pressure field in boundary flows. J. Fluid Mech., Vol. 18, № 2, 1964.
5. Petrovskii, V. S., Hydrodynamic Problems of Turbulent Noise. Leningrad, "Sudostroenie", 1966.

Translated by J. J. D.

UDC 539.3

VARIATIONAL PRINCIPLES OF THE THEORY OF ELASTICITY WITH VARYING INITIAL AND PERTURBED STATES

PMM Vol. 40, № 4, 1976, pp. 752-755

L. Ia. AINOLA
(Tallin)

(Received March 6, 1974)

Two variational principles of Hamilton type are presented for a nonlinear theory of elasticity, which are combined variational principles of the initial and perturbed states of elastic body motion.

Variational formulations of problems to determine the perturbed state of stress for a specified initial linear state are well-known in statics. Variational formulations have also been considered recently for the cases of a nonlinear and time-dependent initial state of stress [1 — 8]. Only quantities in the perturbed state are subjected to variation in the appropriate variational principles.

In order to avoid determining the initial state of stress in the definition of the neutral equilibrium state, varying second-order displacements were additionally